APS Homework 1: Divide-and-Conquer

Problem 1: A Fake among 33 Coins

There are $n = 33$ identical-looking coins. 32 of the coins are genuine and all weigh the same, and 1 coin is fake and weighs slightly less than the genuine coins. Given only a two-pan balance scale (Fig. 1) and the 33 coins, which coin is fake?

Figure 1. A two-pan balance scale.

Problem 1a: What is the minimum number of weighings needed to determine with 100% certainty which of the 33 coins is fake in the worst-case scenario?

Problem 1b: Describe a Divide-and-Conquer algorithm for determining with 100% certainty which of the 33 coins is fake in the minimum number of weighings.

Problem 1c: Generalize the algorithm you provided in *Problem 1b* to work for any arbitrary number of coins $n > 0$.

Problem 1d: Prove that the algorithm you provided in *Problem 1c* is correct for any $n > 0$.

Problem 1e: As a function of *n*, what is the minimum number of weighings needed to determine with 100% certainty which of the *n* coins is fake in the worst-case scenario?

Problem 2: Binary Search

You are given a list *ints* containing $n = |ints| = 8$ integers in ascending order (i.e., 8 integers ordered from smallest to largest). Given an arbitrary integer *x*, does *ints* contain *x*? Define a "comparison" to be a procedure that, given 2 integers *a* and *b*, tells you if $a > b$, $a < b$, or $a = b$.

Problem 2a: What is the minimum number of comparisons needed to determine with 100% certainty if *ints* contains x in the worst-case scenario?

Problem 2b: Describe a Divide-and-Conquer algorithm for determining with 100% certainty if *ints* contains x.

Problem 2c: Generalize the algorithm you provided in *Problem 2b* to work for any sorted list of any arbitrary size $n > 0$.

Problem 2d: Prove that the algorithm you provided in *Problem 2c* is correct for any $n > 0$.

Problem 2e: As a function of n, what is the minimum number of comparisons needed to determine with 100% certainty if an arbitrary sorted list of integers contains x in the worst-case scenario?